Worked Solutions

Edexcel C4 Paper J

1. (*a*) We are given that $\frac{dA}{dt} = 40 \text{ cm}^2 \text{ s}^{-1}$

after 10 seconds area of circle $= 400 \text{ cm}^2$

so $\quad \pi r^2 = 400$

$$r = \sqrt{\frac{400}{\pi}} \text{ cm} \qquad \textbf{(2)}$$

(*b*) $\quad A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

when $r = \sqrt{\frac{400}{\pi}}$ we have, $40 = 2\pi \times \frac{20}{\sqrt{\pi}} \times \frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{1}{\sqrt{\pi}} = 0.56 \text{ cm s}^{-1} \qquad \text{(2 sig. fig.)} \qquad \textbf{(4)}$$

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2. (*a*) $A = \int y \, dx = \int_{t=4}^{t=9} y \frac{dx}{dt} \cdot dt \qquad x = 2t^{\frac{1}{2}}, \quad \frac{dx}{dt} = t^{-\frac{1}{2}}$

$$= \int_4^9 (t^2 - 2)t^{-\frac{1}{2}} \, dt$$

$\qquad \uparrow$ 'y' $\qquad \uparrow$ '$\frac{dx}{dt}$'

$$= \int_4^9 (t^{\frac{3}{2}} - 2t^{-\frac{1}{2}}) dt \qquad \textbf{(4)}$$

(*b*) $A = \left[\frac{2}{5}t^{\frac{5}{2}} - 4t^{\frac{1}{2}}\right]_4^9 = \frac{2}{5} \times 243 - 4 \times 3 - \left(\frac{2}{5} \times 32 - 8\right) = 80.4 \qquad$ **(3)**

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3. (*a*) $\frac{2}{2x-1} + \frac{3}{x+2}$ $\qquad$ (using 'cover up' ...

(*b*) $\int \frac{1}{y} dy = \int \left(\frac{2}{2x-1} + \frac{3}{x+2}\right) dx$

$$\ln y = \ln(2x-1) + 3\ln(x+2) + c$$

$y = 1, x = 1: \qquad \ln 1 = \ln 1 + 3 \ln 3 + c$

$$c = -3 \ln 3$$

$$\therefore \quad \ln y = \ln \frac{(2x-1)(x+2)^3}{27}$$

$$y = \frac{(2x-1)(x+2)^3}{27}$$

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4. (*a*) $f(x) = \left[1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}x^2\right]\left[1 - \frac{1}{2}(-x) + \ldots\right.$

$$= \left(1 + \frac{1}{2}x - \frac{1}{8}x^2\right)\left(1 + \frac{1}{2}x + \frac{3}{8}x^2\right)$$

$$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^2$$

$$= 1 + x + \frac{1}{2}x^2$$

(*b*) valid for $|x| < 1$

(*c*) put $x = \frac{1}{10}$, $\quad f\left(\frac{1}{10}\right) = \sqrt{\frac{1 + \frac{1}{10}}{1 - \frac{1}{10}}} = \sqrt{\frac{11/10}{9/10}} = \ldots$

$\therefore \quad \frac{1}{3}\sqrt{11} \approx 1 + \frac{1}{10} + \left(\frac{1}{2} \times \frac{1}{100}\right) \qquad \sqrt{11} \ldots$

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5. (*a*) (i) putting $B = A$, $\cos(A + A) = \cos A \cos A - \sin A \sin A$

$$= \cos^2 A - \sin^2 A$$
$$= \cos^2 A - (1 - \cos^2 A)$$
$$= 2\cos^2 A - 1 \qquad \textbf{(2)}$$

(ii) $$\int \cos^2 x \ \mathrm{d}x = \frac{1}{2}\int (1 + \cos 2x) \ \mathrm{d}x$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c \qquad \textbf{(3)}$$

(*b*) let $I = \int_0^{\sqrt{3}} \sqrt{4 - x^2} \ \mathrm{d}x$

$$x = 2\sin\theta$$
$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$$
$$x = \sqrt{3}, \quad \theta = \frac{\pi}{3}$$
$$x = 0, \quad \theta = 0$$

$$\therefore \quad I = \int_0^{\frac{\pi}{3}} \sqrt{4 - 4\sin^2\theta} \cdot 2\cos\theta \ \mathrm{d}\theta$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{4(1 - \sin^2\theta)} \cdot 2\cos\theta \ \mathrm{d}\theta = \int_0^{\frac{\pi}{3}} 2\cos\theta \cdot 2\cos\theta \ \mathrm{d}\theta$$

$$= 4\int_0^{\frac{\pi}{3}} \cos^2\theta \ \mathrm{d}\theta$$

$$= \Big[2\theta + \sin 2\theta\Big]_0^{\frac{\pi}{3}} \qquad \text{(from (a) (ii))}$$

$$= \frac{2\pi}{3} + \frac{\sqrt{3}}{2} \qquad \textbf{(5)}$$

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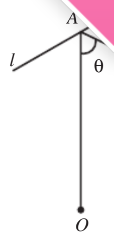
6. (*a*) l passes through A and C.



$$\overrightarrow{CA} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

$$l\text{: } \mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

m passes through B and C

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -5 \\ -7 \end{pmatrix}$$

$$\text{line } m\text{: } \mathbf{r} = \begin{pmatrix} -1 \\ 7 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -5 \\ -7 \end{pmatrix}$$

(*b*) l meets xz-plane where $y = 0$

i.e. where $1 - \lambda = 0$

$$\lambda = 1$$

point of intersection is $\begin{pmatrix} 9 \\ 0 \\ 3 \end{pmatrix}$

(*c*) $\overrightarrow{AO} = \begin{pmatrix} -5 \\ -1 \\ -2 \end{pmatrix}$ $\qquad \overrightarrow{AB} = \begin{pmatrix} -6 \\ 6 \\ 6 \end{pmatrix}$

$|\overrightarrow{AO}| = \sqrt{30}$ $\qquad |\overrightarrow{AB}| = \sqrt{108}$

$AO \cdot AB = 30 - 6 - 12 = 12$

$\therefore \quad 12 = \sqrt{30}\sqrt{108}\cos\theta$

$\theta = 78°$ (nearest degree)

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7. (*a*) $\frac{dy}{dx} = x \cdot \frac{1}{2}(10 - x^2)^{-\frac{1}{2}} \cdot (-2x) + (10 - x^2)^{\frac{1}{2}}$

$$= -\frac{x^2}{\sqrt{10 - x^2}} + \sqrt{10 - x^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow x^2 = 10 - x^2$$

$$x^2 = 5$$

$$x = \sqrt{5}$$

when $x = \sqrt{5}$, $y = \sqrt{5}\sqrt{10 - 5} = 5$

turning point is $(\sqrt{5}, 5)$ **(5)**

(*b*) area $= \int_0^{\sqrt{10}} x\sqrt{10 - x^2}\,dx$ $[y = 0$ at $x = 0,\ \sqrt{10}]$

$$= \left[\frac{-2}{3} \cdot \frac{1}{2}(10 - x^2)^{\frac{3}{2}}\right]_0^{\sqrt{10}}$$ [or let $u = 10 - x^2$]

$$= -\frac{1}{3} \times 0 - \left(-\frac{1}{3} \cdot 10^{\frac{3}{2}}\right)$$

$$= \frac{10}{3}\sqrt{10}$$ **(7)**

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8. (*a*) $\frac{dN}{dt} = -kN$

(*b*) $\int \frac{1}{N}\,dN = -\int k\ dt$

$$\ln N = -kt + \ln A$$

$$\ln \frac{N}{A} = -kt$$

$$\frac{N}{A} = e^{-kt}$$

$$N = Ae^{-kt}$$

(*c*) $t = 0,\ N = 5 \times 10^{12}$

$\therefore\quad A = 5 \times 10^{12}$

$t = 10,\ N = 2 \times 10^{11}$

$$2 \times 10^{11} = 5 \times 10^{12}e^{-10k}$$

$$\frac{1}{25} = e^{-10k}$$

$$\ln \frac{1}{25} = -10k$$

$$k = -\frac{1}{10}\ln\frac{1}{25} = \frac{1}{10}\ln 25 = 0.32188\ldots$$

(*d*) when $t = 40$, $N = 5 \times 10^{12} \times e^{-0.32188 \times 40}$

$N = 1.28 \times 10^7$ (3 sig. fig.)
